Gravitational Phase Transition of Fermionic Matter in a General-relativistic Framework

LIVING SYSTEMS RESEARCH

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LABORATOIRE DE PHYSIQUE THÉORIQUE TOULOUSE

Abstract

The Thomas-Fermi model at finite temperature is extended to describe a system of self-gravitating massive fermions in a general-relativistic framework. It is shown that, in the canonical ensemble, when a nondegenerate fermionic gas is cooled below a critical temperature a condensed phase emerges, consisting of quasi-degenerate fermion stars. In the microcanonical ensemble, similarly, a condensed phase emerges when the energy of the system is below a critical value.

Theoretical Framework

We consider a (static) fermionic gas, formed by N particles of mass m placed within a spherical box of dimension R. Here we neglect the contributions from other interactions. To study the nature of the phase transitions, we first solve the Tolman-Oppenheimer-Volkoff (TOV) system

$$\frac{d\Phi}{dr} = -\frac{2G}{c^4}\frac{(\Phi+1)(M_r c^2 + 4\pi P r^3)}{r^2}\left(1-\frac{2GM_r}{rc^2}\right)^{-1}$$

$$\frac{dM_r}{dr} = \frac{4\pi\epsilon r^2}{c^2}$$

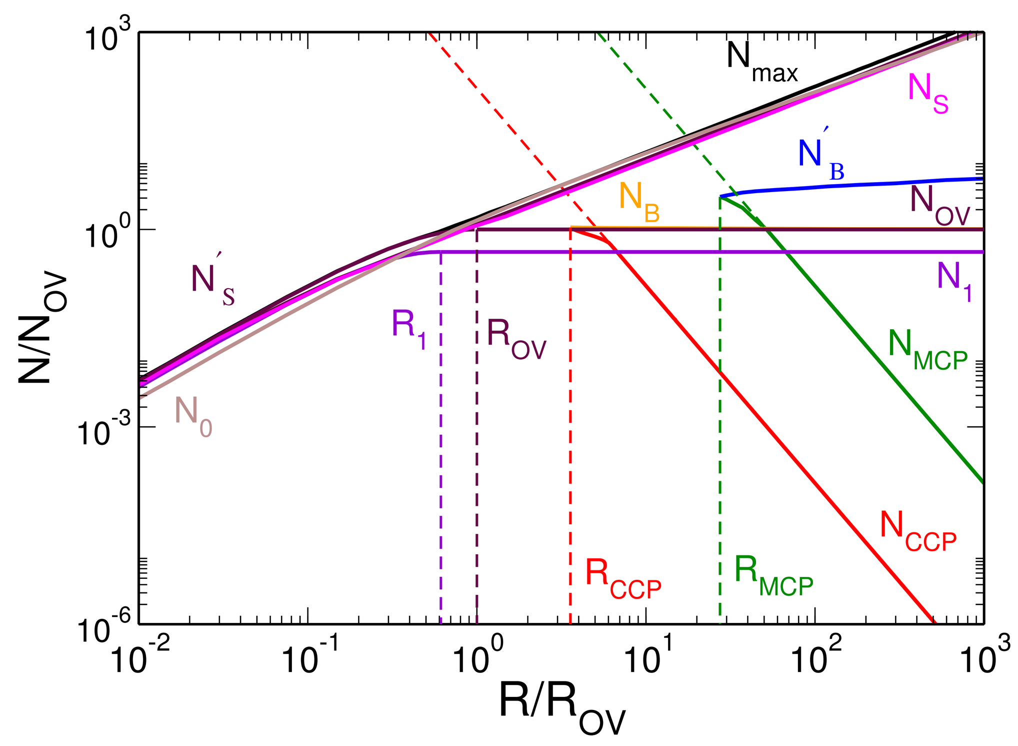
(supplemented by the conditions $\Phi(0) = \Phi_0$ and $M_r(0) = 0$) for several values of Φ_0 (the central value of the gravitational potential). Then, for a given value of the baryon number

$$N = N(\Phi_0) = \int_0^R 4\pi n r^2\left(1-\frac{2GM_r}{rc^2}\right)^{-1/2} dr$$

we can plot the series of equilibria. We define the variables

$$\Lambda = -\frac{E_b R}{GN^2m^2} = \frac{(Nm-M)Rc^2}{GN^2m^2}, \qquad \eta = \frac{GNm^2}{k_B T R}$$

The occurrence of the phase transition depends on characteristic values of N and R (expressed in units of N_{OV} and R_{OV}, OV = Oppenheimer-Volkoff limit). R_{CCP}: Canonical Critical Radius; $R_{MCP} > R_{CCP}$: Microcanonical Critical Radius. N_{CCP}: Canonical Critical Point; $N_{MCP} > N_{CCP}$: Microcanonical Critical Point.



Formation of Structures

Here we give two examples of astrophysical applications. The starting configurations are characterized by R and M. For the Dark Matter (DM) Fermions Balls we have used $m = 17.2 keV/c^2$.

Canonical Ensemble

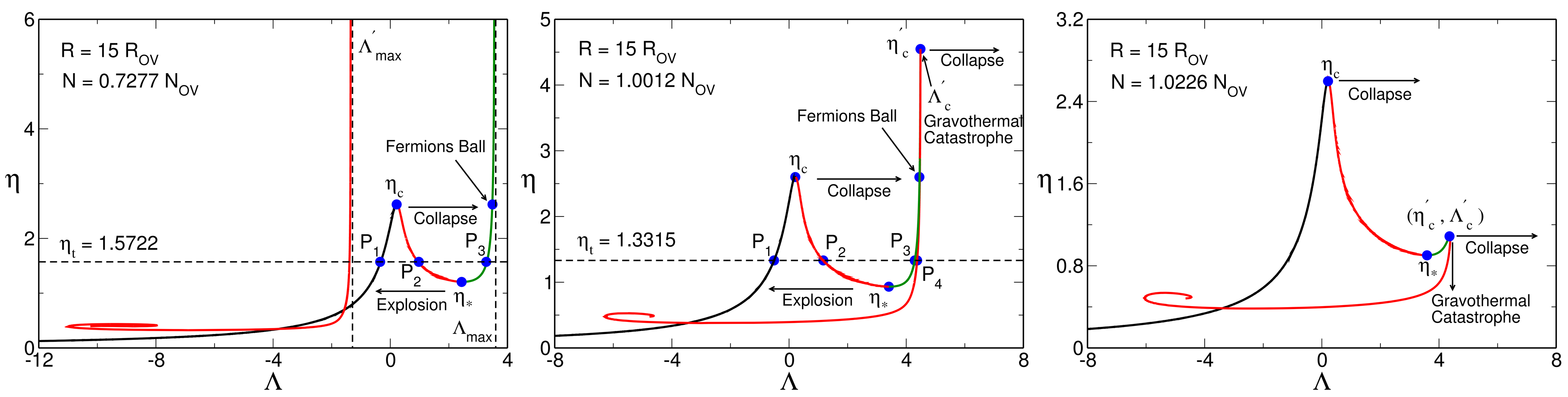
	Neutron Star	DM Fermions Ball
R	2.73×10^{2} km	2.64×10^{-2} pc
R_{CCP}	3.60×10^{1} km	3.49×10^{-3} pc
R_{cond}	1.12×10^{1} km	1.08×10^{-3} pc
M	$7.20\times10^{-1}\ M_\odot$	$2.10\times10^{9}\ M_\odot$
M_{cond}	$6.79\times10^{-1}\ M_\odot$	$2.0293\times10^{9}\ M_\odot$
T_t	4.79×10^{10} K	8.77×10^{5} K
T_{cond}	1.64×10^{10} K	3.01×10^{5} K
E_{gas}	-9.49×10^{50} erg	-2.83×10^{60} erg
E_{min}	-4.05×10^{52} erg	-1.21×10^{62} erg
t_D	1.48×10^{-2} s	1.40 yrs

Microcanonical Ensemble

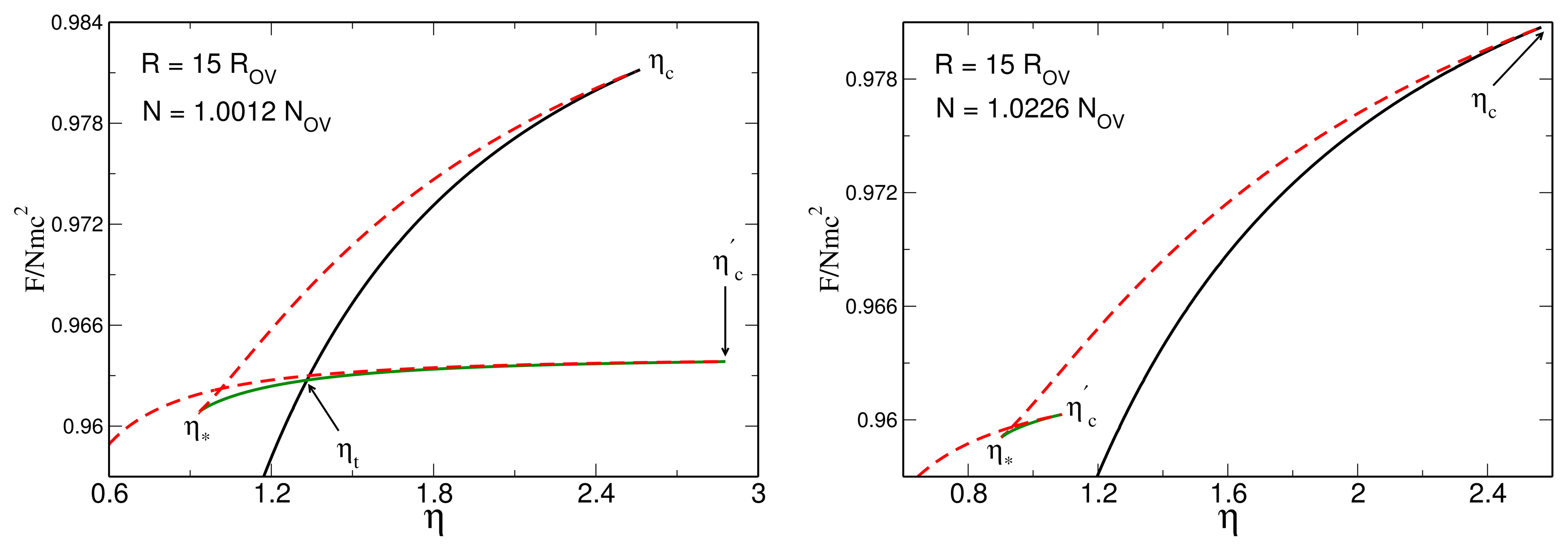
	Neutron Star	DM Fermions Ball
R	4.88×10^{3} km	3.15×10^{2} pc
R_{MCP}	4.35×10^{2} km	2.81×10^{1} pc
R_{cond}	2.50×10^{1} km	1.62 pc
M	$1\ M_\odot$	$10\ M_\odot$
M_{cond}	$2.20\times10^{-1}\ M_\odot$	$2.20\ M_\odot$
T_{gas}	1.63×10^{9} K	1.49×10^{-7} K
T_{cond}	1.14×10^{10} K	1.05×10^{-6} K
E_{coll}	-1.81×10^{50} erg	-9.10×10^{39} erg

$R = 15\ R_{OV}$: Canonical Instabilities

Equilibrium phase diagrams for fermionic systems in GR. η_t is the temperature where the phase transition occurs. **Left**: At the temperature η_c the system evolves from the gaseous (black full line) to the condensed phase (green full line). The gravitational collapse is prevented by Pauli's exclusion principle. At the temperature η_*, the system evolves from the condensed phase to the gaseous one. The gravitational explosion is halted by the box. **Centre**: The condensed phase collapses towards a Black Hole at the temperature η'_c. **Right**: The transition phase is suppressed.

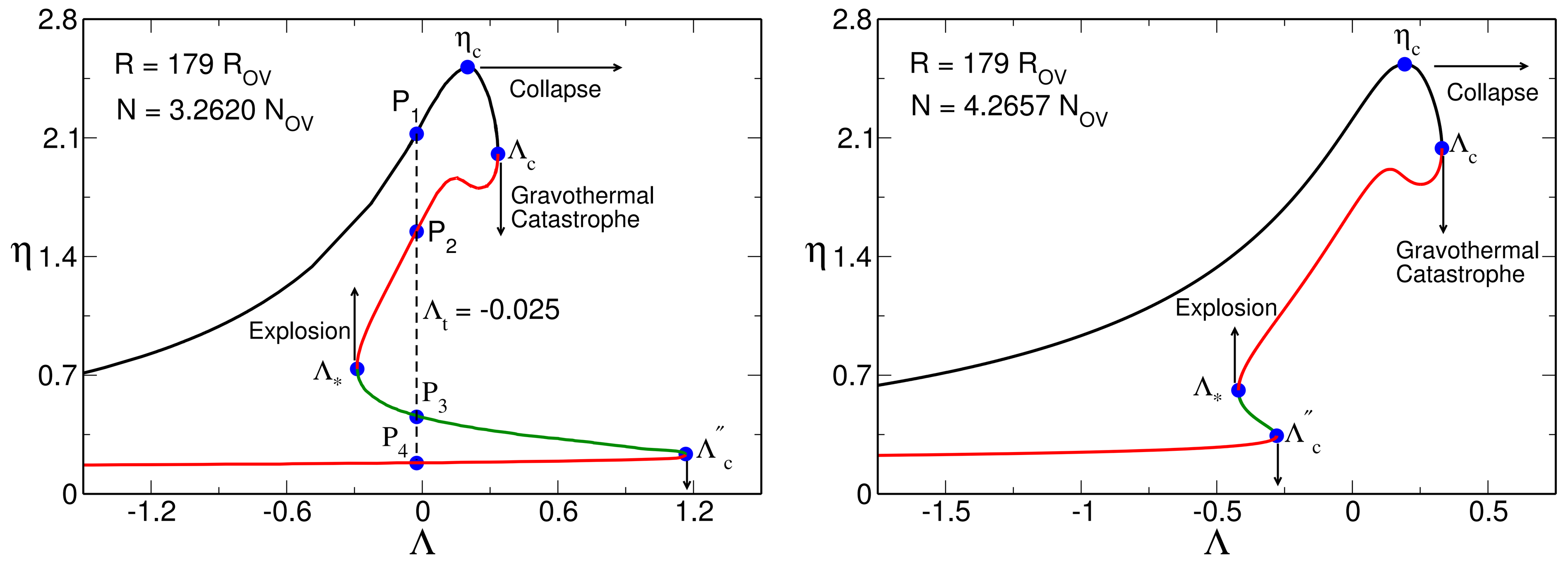


Free energy as a function of the normalized inverse temperature η. **Left**: The phase transition is identified by the intersection point between the gaseous (black solid line) and the condensed branch (green solid line). **Right**: The intersection point between the gaseous and the condensed phase is missing, so the phase transition is suppressed (the crossing point concerns unstable states).



$R = 179\ R_{OV}$: Microcanonical Instabilities

Equilibrium phase diagrams for fermionic systems in GR. **Left**: A phase transition from the gaseous to the condensed phase occurs at the energy Λ_t. At the energy Λ_c the system evolves from the gaseous to the condensed phase. The gravitational collapse is prevented by quantum degeneracy. At the energy Λ_* the system evolves from the condensed to the gaseous phase. The gravitational explosion is halted by the box. At the energy Λ'_c the system collapses towards a Black Hole. **Right**: The phase transition is suppressed.



Entropy as a function of the normalized energy Λ. **Left**: The phase transition is identified by the intersection point between the gaseous (black solid line) and the condensed branch (green solid line). **Right**: The intersection point between the gaseous and the condensed phase is missing, so the phase transition is suppressed (the crossing point concerns unstable states).