# A Symbol Dynamic Approach to Characterize the Chaoticity of a Time Series

**Florian Wodlei***, Mihnea R. Hristea

Living Systems RESEARCH, Klagenfurt, AUSTRIA, www.ilsr.at, *florian.wodlei@ilsr.at
Interdisciplinary research cluster on Self-Organizing Systems, University Klagenfurt, https://selforganizing.wordpress.com/

*Oscillations and Dynamic Instabilities in Chemical Systems (GRS)*
*Gordon Research Seminar, Les Diablerets, July 13-14, 2024*

***ABSTRACT.*** *Determination of the behavior of a system by analyzing the dynamics of one or more variables characterizing the system over time is a canonical approach in the scientific context.*
*The evaluation of such time series is based on a number of different methods such as the calculation of the Lyapunov-Exponent, the phase space reconstruction by time delay or the Fast Fourier Transform (FFT) and the corresponding power spectrum.*
*Each of these methods have their limitations and create more or less reliable results. For highly complex systems, as, for example, the Belousov Zhabotinsky reaction, conclusive results are difficult to obtain.*
*Neither the transcription of time series into sequences of symbols nor the statistical analysis of these sequences is new. What is new in our approach is that we are not only looking at the topological invariants but also at the detailed distributions of the frequencies of different words that correspond to each times series sequence. This approach allows us to describe the different types of behavior, specifically the chaotic one, in a more accurate way than the above mentioned methods.*

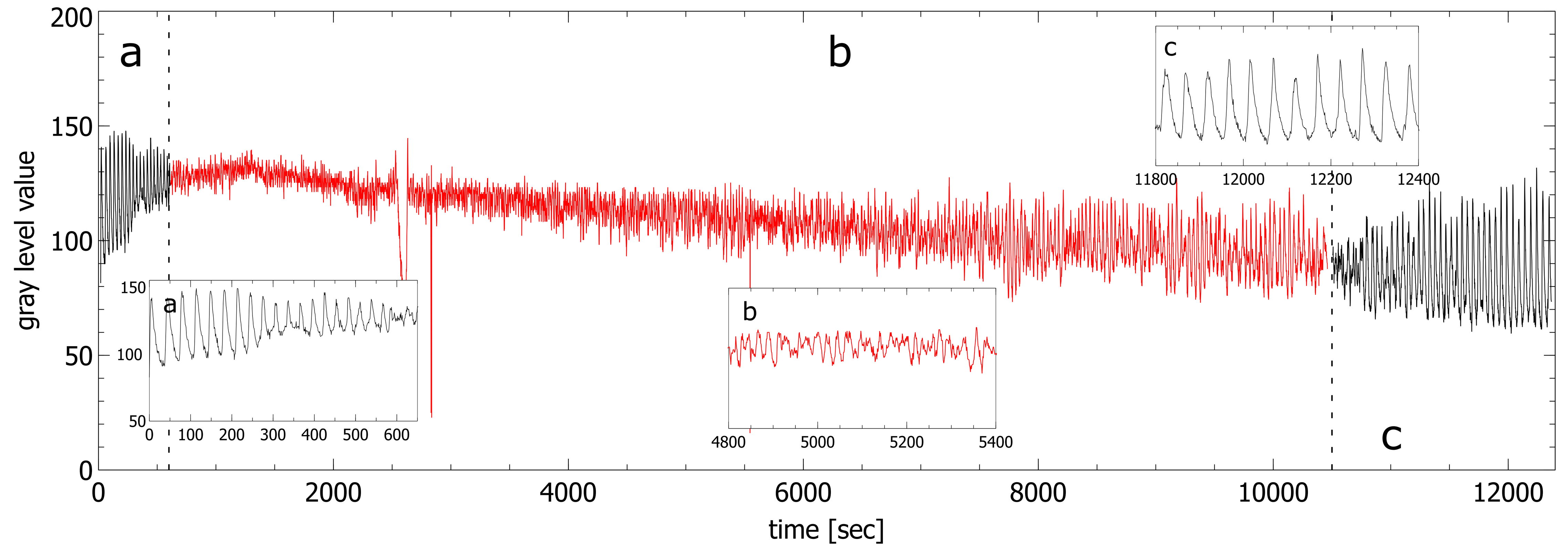


Figure 1: Oscillations of the concentration of ferriin in the Belousov Zhabotisnky reaction (spikes in the time series correspond to the passage of bubbles). The distinct phases a, b, and c represent the a periodic (a), a so called chaotic (b) and a second periodic (c) phase in the time evolution of the system. Insets show magnifications of these phases.

## Introduction

To determine whether a time series is periodic or not, is not always an easy task, specially when this time series is the result of a complex interplay between different physico-chemical processes, as it is the case in the Belousov Zhabotinsky (BZ) reaction (time series given in Fig. 1).
In an earlier work we discovered periodic and ordered structures within the so-called "*chaotic*" phase of the BZ reaction [1].
The ordered structures (convection rolls) we discovered took place at the same moment where the color of the system was locally changing in an aperiodic (chaotic) manner.

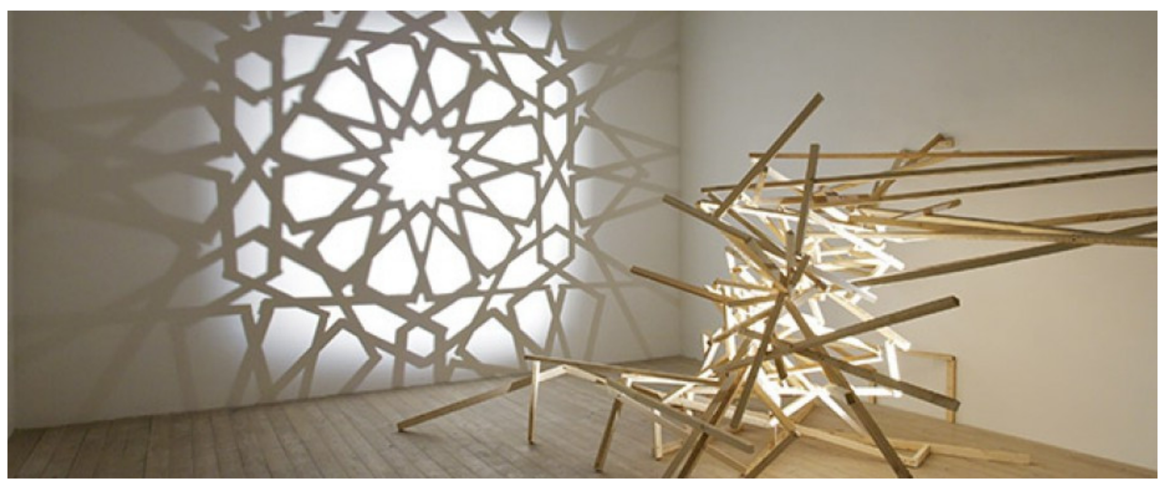

Figure 2: Artistic installation by Azerbaijan artist Rashad Alakbarov on Chaos.

Looking at the hydrodynamics one sees *order* and looking on the local color change one sees *chaos*. The artist Rashad Alakbarov shows with his art a similar phenomena (see Fig. 2).
Interestingly, the FFT spectra of the corresponding chaotic phase (see Fig.3) and the Ljapunov exponent (λ = 0.0014 ± 0.00027) showed a clear signature of chaos, independently of the hydrodynamic order.

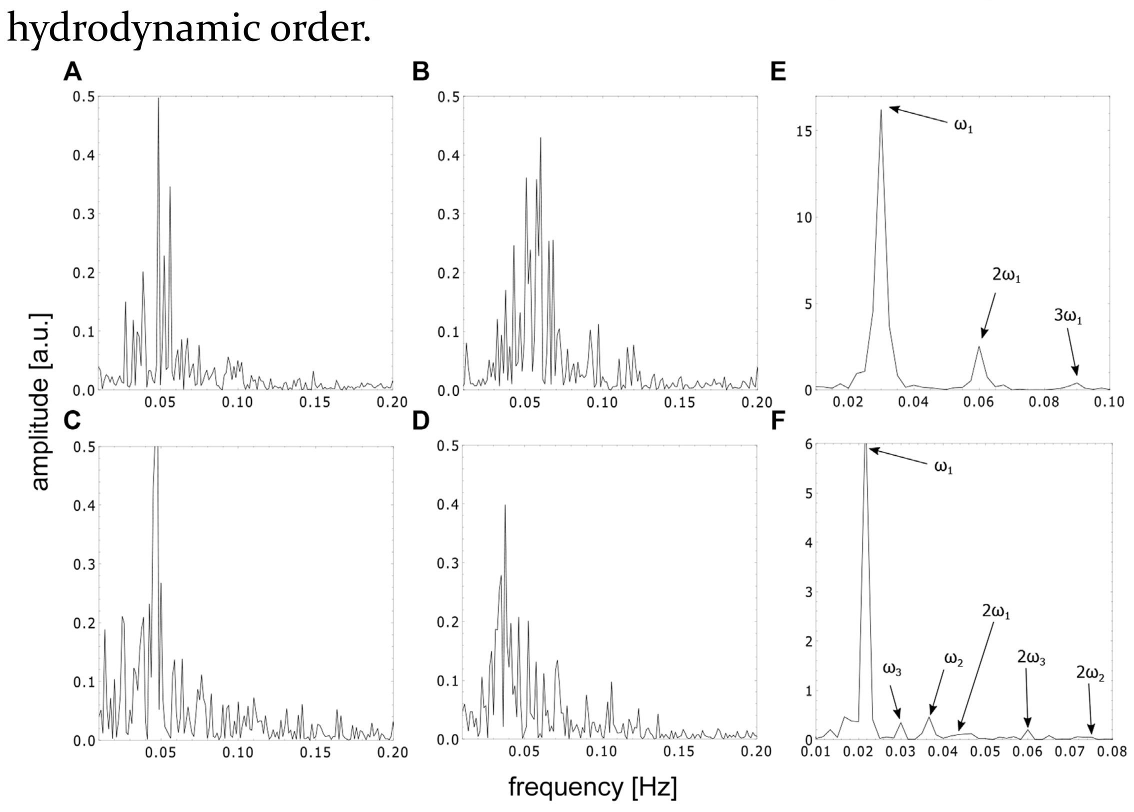


Figure 3: FFT spectra of the different phases in the BZ reaction. (A,B,C,D) show spectra of different moments in the chaotic phase. (E and F) show the spectra of the first and the second periodic phase, respectively.

The fact that these methods do not reflect the hydrodynamic order in the chaotic phase motivated us to investigate another way of characterizing the chaoticity of a time series.
In this approach we followed an idea by G. Nicolis and I. Prigogine [2]. According to Fig. 4 a time series can be "*translated*" into a *sequence of symbols* of a given alphabet.
The alphabet used in this approach is {L,R} for left and right. The corresponding sequence of symbols of a given length $N$ can be understood to consist of an array of possible words of different length $L$. By investigating the frequency of all possible words of length $L$ that can be found in the whole sequence on can learn something about the periodicity of the sequence.

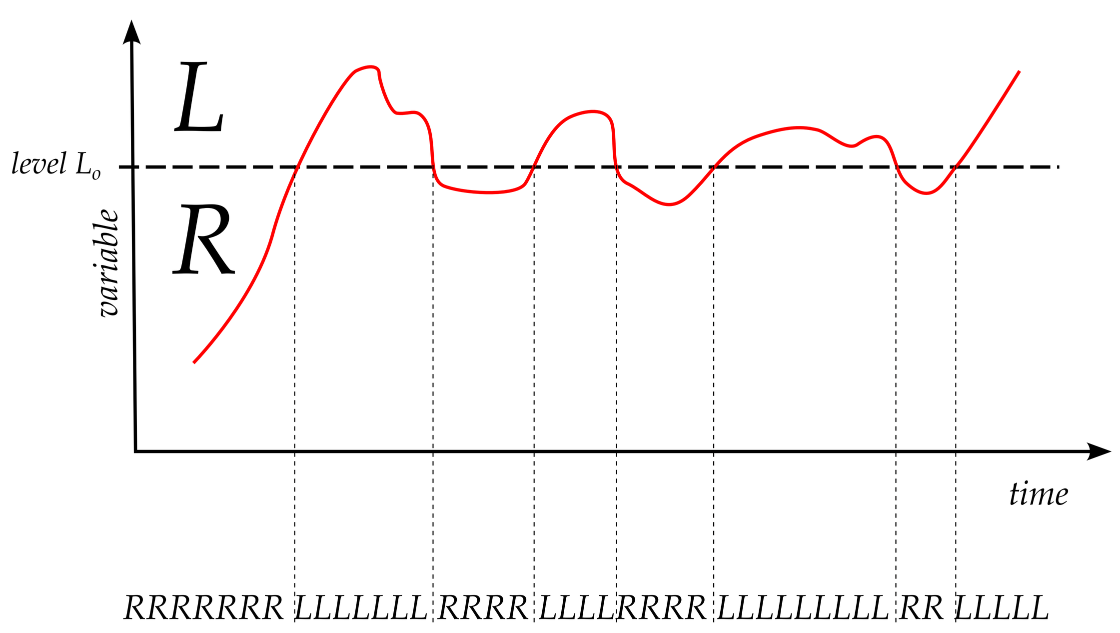


Figure 4: Method of translating a time series into a sequence of symbols

If a sequence is random the distribution of frequencies for a given word length is uniformly distributed. In other words, all possible words would have the same probability.
On the other hand, any deviation from this equiprobably distribution would show the rise of periodicity in the sequence.

A detailed study showed us that the distribution of these frequencies can give a much more detailed inside in the character of the time series than obtained by the methods mentioned before. Fig. 5 gives a detailed picture on the different types of distributions for the different types of word sequences.

## Results and Discussion

The approach described in this work gave us new insides in the chaotic regime of the BZ reaction and might also help to get better insides in other time series corresponding to complex dynamics in other fields of research.
Nevertheless, still a lot of work needs to be done to fully understand the different types distributions and moreover to interpret them correctly.

## Literature

[1] Wodlei F, Hristea M and Alberti G, 2022, Front. Chem. 10
[2] Nicolis G, Prigogine I, 1989 , W.H. Freeman & Co.

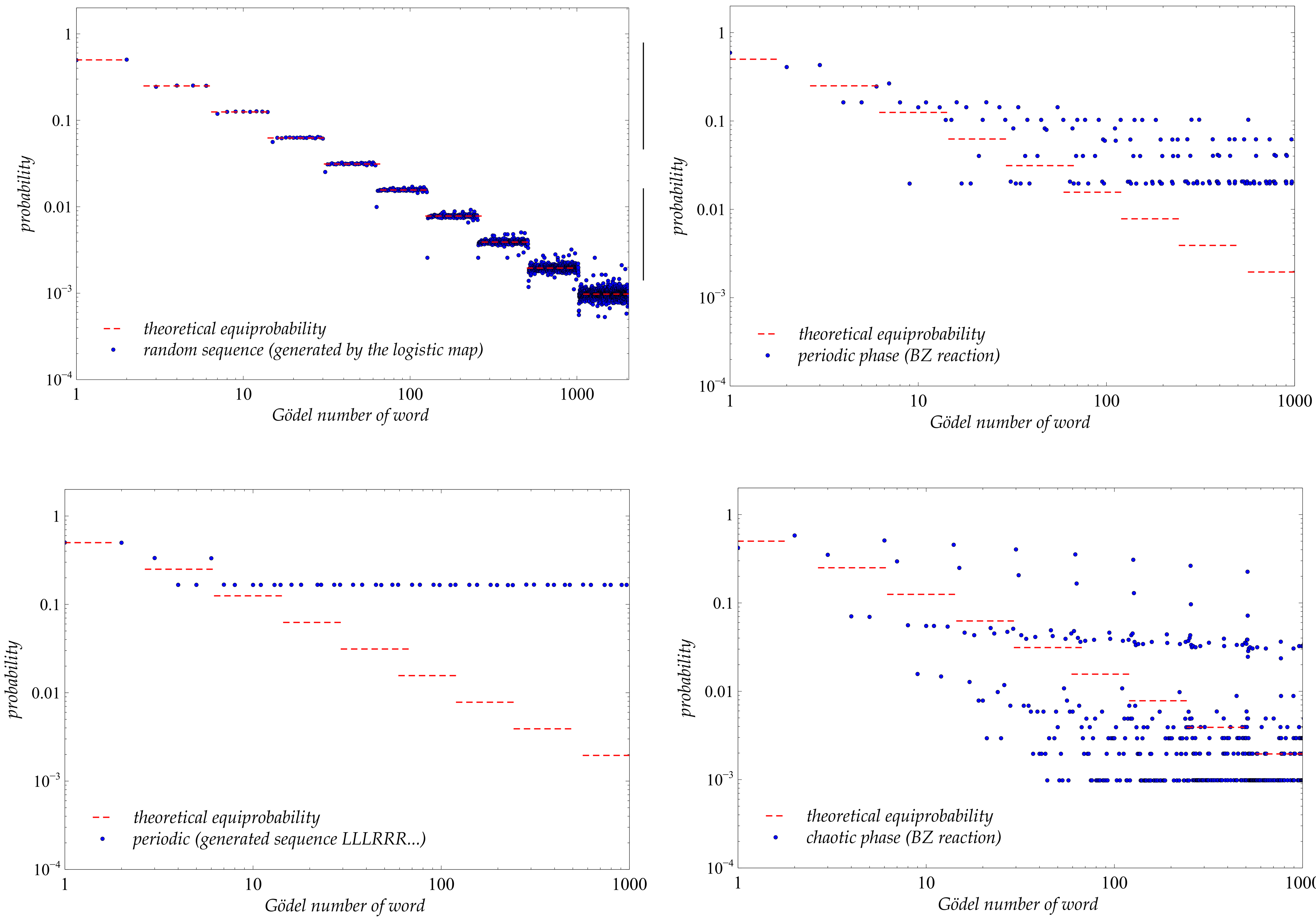


Figure 5: Probability distribution for different word sequences. (A) shows a distribution generated by the logistic map in the choatic regime (λ = 3.9999). (B) shows a distribution of a perfect periodic sequence LLLRRR... (C) shows the distribution of the periodic phase. (D) shows the distribution of the chaotic phase.