

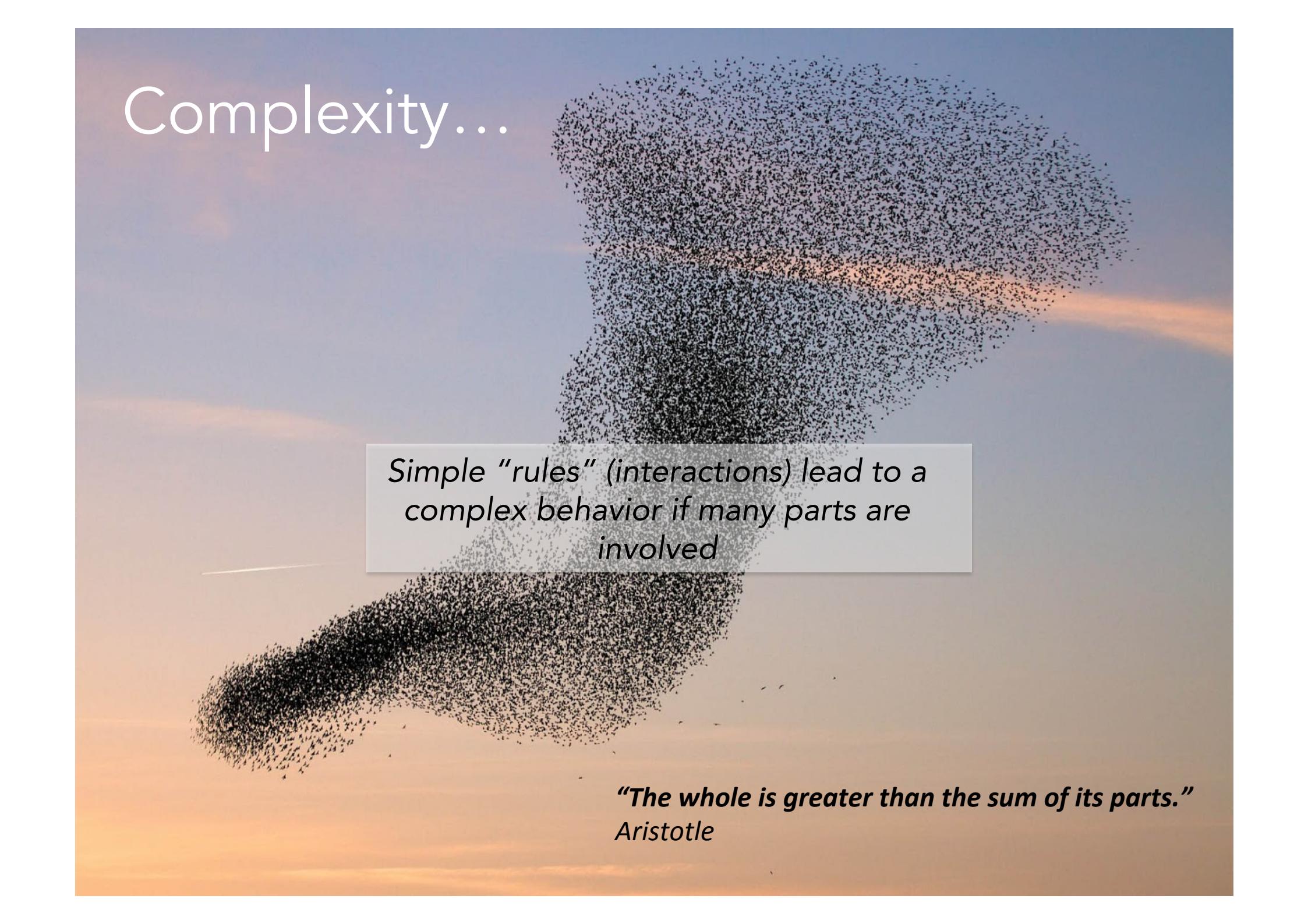
Florian Wodlei and Mihnea R Hristea

# Hydrodynamic Control of Chaos in a Belousov Zhabotinsky Oscillator

Conference on Complex Systems 2018  
Thessaloniki Greece  
23-28 September 2018



# Complexity...

A photograph of a massive flock of birds, likely starlings, captured in flight against a backdrop of a sunset or sunrise. The birds are concentrated in the upper right quadrant, forming a dense, dark cloud against a lighter sky. The sky transitions from a deep blue at the top to warm orange and yellow hues near the horizon. A single, thin white contrail or streak of light cuts across the upper left portion of the image.

Simple “rules” (interactions) lead to a complex behavior if many parts are involved

***“The whole is greater than the sum of its parts.”***  
Aristotle

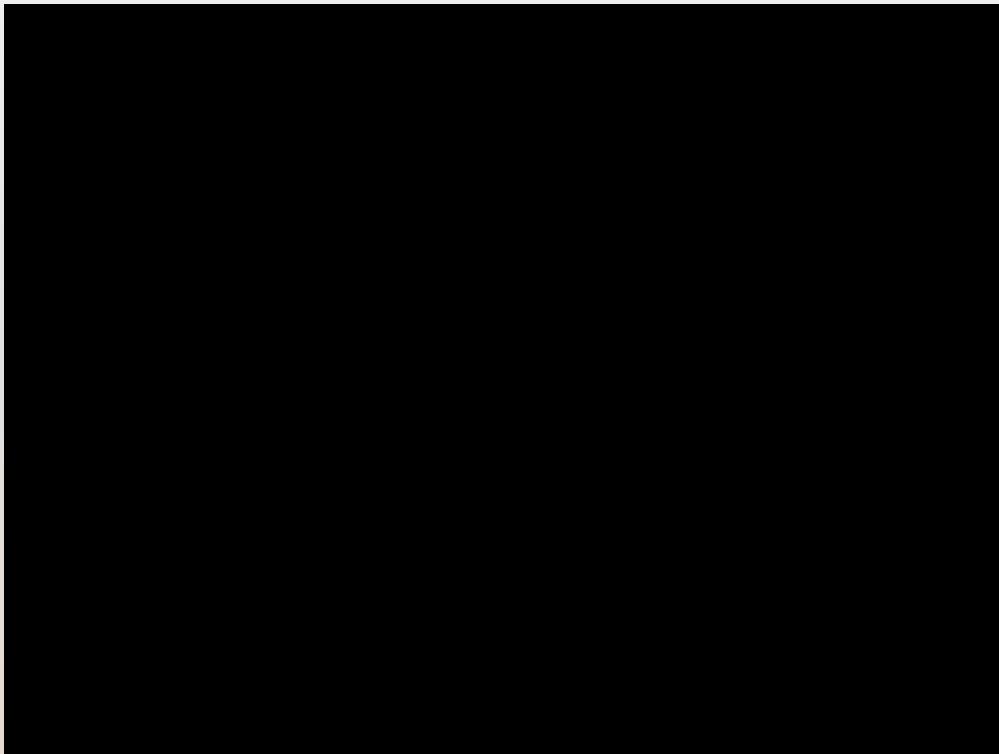
# Chaos...

*Something is chaotic if we cannot find an order*

*Deterministic  
Chaos: sensitivity  
to initial conditions*

Greek **χάος** means "emptiness, vast void,  
chasm, abyss"

# The Belousov Zhabotinsky Reaction *[Bjela'usov-Shabatinski]*



Boris P. Belousov



Anatol M. Zhabotinsky

*“Cyclic (periodic) bromination of an organic species in acidic environment with the help of a catalyst”*

1. Неорганическая часть				
1	$\text{HOBr} + \text{Br}^\cdot + \text{H}^+ \rightarrow \text{Br}_2 + \text{H}_2\text{O}$	$2.3 \cdot 10^9$	8	$\text{BrO}_3^- + \text{HOBr} + \text{H}^+ \rightarrow 2\text{HBrO}_2$
2	$\text{Br}_2 + \text{H}_2\text{O} \rightarrow \text{HOBr} + \text{Br}^\cdot + \text{H}^+$	2.0	9	$\text{BrO}_3^- + \text{HBrO}_2 + \text{H}^+ \rightarrow \text{Br}_2\text{O}_4 + \text{H}_2\text{O}$
3	$\text{Br}^\cdot + \text{HBrO}_2 + \text{H}^+ \rightarrow 2\text{HOBr}$	$2.0 \cdot 10^6$	10	$\text{Br}_2\text{O}_4 + \text{H}_3\text{O} \rightarrow \text{BrO}_3^- + \text{HBrO}_2 + \text{H}^+$
4	$2\text{HOBr} \rightarrow \text{Br}^\cdot + \text{HBrO}_2 + \text{H}^+$	$2.0 \cdot 10^5$	11	$\text{Br}_2\text{O}_4 \rightarrow 2\text{BrO}_2^\cdot$
5	$\text{Br}^\cdot + \text{HBrO}_2 + 2\text{H}^+ \rightarrow \text{HOBr} + \text{HBrO}_2$	2.0	12	$2\text{BrO}_2^\cdot \rightarrow \text{Br}_2\text{O}_4$
6	$\text{HOBr} + \text{HBrO}_2 \rightarrow \text{Br}^\cdot + \text{HBrO}_3^\cdot + 2\text{H}^+$	3.3	13	$\text{Ce}^{3+} + 2\text{BrO}_2^\cdot + \text{H}^+ \rightarrow \text{HBrO}_2 + \text{Ce}^{4+}$
7	$2\text{HBrO}_2 \rightarrow \text{BrO}_3^- + \text{HOBr} + \text{H}^+$	$3.0 \cdot 10^3$	14	$\text{HBrO}_2 + \text{Ce}^{4+} \rightarrow \text{Ce}^{3+} + 2\text{BrO}_2^\cdot + \text{H}^+$
2. Реакции с участием органических веществ				
а) Реакции без участия и образования радикалов				
15	$\text{MA} \rightarrow \text{ENOL}$	$3.0 \cdot 10^{-3}$	20	$\text{TTA} + \text{HOBr} \rightarrow \text{BrTTA} + \text{H}_2\text{O}$
16	$\text{ENOL} \rightarrow \text{MA}$	200.0	21	$\text{BrO}_3\text{MA} + \text{H}_2\text{O} \rightarrow \text{HBrO}_2 + \text{TTA}$
17	$\text{ENOL} + \text{Br}_2 \rightarrow \text{BrMA} + \text{Br}^\cdot + \text{H}^+$	$1.91 \cdot 10^6$	22	$\text{BrO}_3\text{MA} \rightarrow \text{HOBr} + \text{MOA}$
18	$\text{MA} + \text{HOBr} \rightarrow \text{Br}_2\text{MA} + \text{H}_2\text{O}$	8.2	23	$\text{BrO}_3\text{TTA} \rightarrow \text{HBrO}_2 + \text{MOA}$
19	$\text{Br}_2\text{MA} + \text{H}_2\text{O} \rightarrow \text{MA} + \text{HOBr}$	0.1	24	$\text{BrTTA} \rightarrow \text{Br}^\cdot + \text{MOA} + \text{H}^+$
б) Реакции с образованием радикалов				
25	$\text{Ce}^{4+} + \text{BrMA} \rightarrow \text{Ce}^{3+} + \text{BrMA}^\cdot + \text{H}^+$	0.09	29	$\text{Ce}^{4+} + \text{MOA} + \text{H}_2\text{O} \rightarrow \text{Ce}^{3+} + \text{OA}^\cdot + \text{COOH} + \text{H}^+$
26	$\text{Ce}^{4+} + \text{MA} \rightarrow \text{Ce}^{3+} + \text{MA}^\cdot + \text{H}^+$	0.23	30	$\text{HOBr} + \text{OA}^\cdot \rightarrow \text{Br}^\cdot + \text{COOH} + \text{CO}_2 + \text{H}_2\text{O}$
27	$\text{Ce}^{4+} + \text{TTA} \rightarrow \text{Ce}^{3+} + \text{TTA}^\cdot + \text{H}^+$	0.66	31	$\text{Ce}^{4+} + \text{OA}^\cdot \rightarrow \text{Ce}^{3+} + \text{COOH} + \text{H}^+ + \text{CO}_2 + \text{H}^+$
28	$\text{HOBr} + \text{MOA} \rightarrow \text{Br}^\cdot + \text{OA}^\cdot + \text{COOH}$	140.0	32	$\text{BrO}_3^- + \text{OA}^\cdot + \text{H}^+ \rightarrow \text{BrO}_2^\cdot + \text{COOH} + \text{CO}_2 + \text{H}^+$
в) Реакции гибели радикалов				
33	$2\text{Br}^\cdot \rightarrow \text{Br}_2$	$1.0 \cdot 10^8$	44	$\text{MA}^\cdot + \text{Br}^\cdot \rightarrow \text{BrMA}$
34	$\text{Br}^\cdot + \text{BrMA}^\cdot \rightarrow \text{Br}_2\text{MA}$	$1.0 \cdot 10^9$	45	$\text{MA}^\cdot + \text{Ce}^{3+} + \text{H}^+ \rightarrow \text{MA} + \text{Ce}^{4+}$
35	$2\text{BrMA}^\cdot + \text{H}_2\text{O} \rightarrow \text{BrMA} + \text{BrTTA}$	$1.0 \cdot 10^8$	46	$\text{MA}^\cdot + \text{BrO}_2^\cdot \rightarrow \text{BrO}_2\text{MA}$
36	$\text{BrMA}^\cdot + \text{MA} + \text{H}_2\text{O} \rightarrow \text{MA} + \text{BrTTA}$	$1.0 \cdot 10^9$	47	$2\text{TTA}^\cdot \rightarrow \text{TTA} + \text{MOA}$
37	$\text{BrMA}^\cdot + \text{TTA}^\cdot + \text{H}_2\text{O} \rightarrow \text{TTA} + \text{BrTTA}$	$1.0 \cdot 10^9$	48	$\text{TTA}^\cdot + \text{COOH} \rightarrow \text{TTA} + \text{CO}_2$
38	$\text{BrMA}^\cdot + \text{Ce}^{3+} + \text{H}_2\text{O} \rightarrow \text{Ce}^{3+} + \text{BrTTA} + \text{H}^+$	$1.0 \cdot 10^7$	49	$\text{TTA}^\cdot + \text{Br}^\cdot \rightarrow \text{BrTTA}$
39	$\text{BrMA}^\cdot + \text{BrO}_3^- + \text{H}_2\text{O} \rightarrow \text{HBrO}_2 + \text{BrTTA}$	$5.0 \cdot 10^9$	50	$\text{TTA}^\cdot + \text{Ce}^{3+} + \text{H}^+ \rightarrow \text{TTA} + \text{Ce}^{4+}$
40	$\text{BrMA}^\cdot + \text{COOH} \rightarrow \text{BrMA} + \text{CO}_2$	$5.0 \cdot 10^8$	51	$\text{TTA}^\cdot + \text{BrO}_2^\cdot \rightarrow \text{BrO}_2\text{TTA}$
41	$2\text{MA}^\cdot + \text{H}_2\text{O} \rightarrow \text{MA} + \text{TTA}$	$3.2 \cdot 10^9$	52	$2\text{COOH} \rightarrow \text{OA}$
42	$\text{MA}^\cdot + \text{TTA}^\cdot + \text{H}_2\text{O} \rightarrow 2\text{TTA}$	$1.0 \cdot 10^9$	53	$^*\text{COOH} + \text{Ce}^{4+} \rightarrow \text{Ce}^{3+} + \text{CO}_2 + \text{H}^+$
43	$\text{MA}^\cdot + \text{COOH} \rightarrow \text{MA} + \text{CO}_2$	$2.0 \cdot 10^9$	54	$^*\text{COOH} + \text{Br}^\cdot \rightarrow \text{Br}^\cdot + \text{CO}_2 + \text{H}^+$
			55	$^*\text{COOH} + \text{BrO}_2^\cdot \rightarrow \text{HBrO}_2 + \text{CO}_2$
г) Реакции продолжения цепи				
56	$\text{MA}^\cdot + \text{Br}_2 \rightarrow \text{BrMA} + \text{Br}^\cdot$	$1.5 \cdot 10^8$	69	$\text{BrMA}^\cdot + \text{HOBr} \rightarrow \text{BrTTA} + \text{Br}^\cdot$
57	$\text{MA}^\cdot + \text{HOBr} \rightarrow \text{TTA} + \text{Br}^\cdot$	$1.0 \cdot 10^7$	70	$\text{BrMA}^\cdot + \text{BrO}_3^- + \text{H}^+ \rightarrow \text{BrTTA} + \text{BrO}_2^\cdot$
58	$\text{MA}^\cdot + \text{BrO}_3^- + \text{H}^+ \rightarrow \text{TTA} + \text{BrO}_2^\cdot$	40.0	71	$^*\text{COOH} + \text{BrMA} \rightarrow \text{Br}^\cdot + \text{MA}^\cdot + \text{CO}_2 + \text{H}^+$
59	$\text{MA}^\cdot + \text{TTA} \rightarrow \text{MA} + \text{TTA}$	$1.0 \cdot 10^5$	72	$^*\text{COOH} + \text{Br}_2 \rightarrow \text{Br}^\cdot + \text{Br}^\cdot + \text{CO}_2 + \text{H}^+$
60	$\text{TTA}^\cdot + \text{MA} \rightarrow \text{MA}^\cdot + \text{TTA}$	$1.0 \cdot 10^5$	73	$^*\text{COOH} + \text{HOBr} \rightarrow \text{Br}^\cdot + \text{CO}_2 + \text{H}_2\text{O}$
61	$\text{MA}^\cdot + \text{BrMA} \rightarrow \text{MA} + \text{BrMA}^\cdot$	$1.0 \cdot 10^5$	74	$^*\text{COOH} + \text{BrO}_3^- + \text{H}^+ \rightarrow \text{BrO}_2^\cdot + \text{CO}_2 + \text{H}_2\text{O}$
62	$\text{BrMA}^\cdot + \text{MA} \rightarrow \text{BrMA} + \text{MA}^\cdot$	$5.0 \cdot 10^2$	75	$\text{Br}^\cdot + \text{MA} \rightarrow \text{Br}^\cdot + \text{MA}^\cdot + \text{H}^+$
63	$\text{TTA}^\cdot + \text{BrMA} \rightarrow \text{TTA} + \text{BrMA}^\cdot$	$2.0 \cdot 10^5$	76	$\text{Br}^\cdot + \text{TTA} \rightarrow \text{Br}^\cdot + \text{TTA}^\cdot + \text{H}^+$
64	$\text{BrMA}^\cdot + \text{TTA} \rightarrow \text{BrMA} + \text{TTA}^\cdot$	$5.0 \cdot 10^3$	77	$\text{Br}^\cdot + \text{BrMA} \rightarrow \text{Br}^\cdot + \text{BrMA}^\cdot + \text{H}^+$
65	$\text{TTA}^\cdot + \text{Br}_2 \rightarrow \text{BrTTA} + \text{Br}^\cdot$	$1.0 \cdot 10^8$	78	$\text{Br}^\cdot + \text{MOA} + \text{H}_2\text{O} \rightarrow \text{Br}^\cdot + \text{OA}^\cdot + \text{COOH} + \text{H}^+$
66	$\text{TTA}^\cdot + \text{HOBr} \rightarrow \text{MOA} + \text{Br}^\cdot + \text{H}_2\text{O}$	$1.0 \cdot 10^7$	79	$\text{Br}^\cdot + \text{OA}^\cdot \rightarrow \text{Br}^\cdot + \text{COOH} + \text{CO}_2 + \text{H}^+$
67	$\text{TTA}^\cdot + \text{BrO}_3^- + \text{H}^+ \rightarrow \text{MOA} + \text{BrO}_2^\cdot + \text{H}_2\text{O}$	40.0	80	$\text{BrO}_2^\cdot + \text{OA}^\cdot \rightarrow \text{HBeO}_2 + \text{COOH} + \text{CO}_2$
68	$\text{BrMA}^\cdot + \text{Br}_2 \rightarrow \text{Br}_2\text{MA} + \text{Br}^\cdot$	$1.0 \cdot 10^6$		

80-stage reaction mechanism of the Belousov-Zhabotinsky reaction from

Gyorgyi L., Turanyi T., Field R.J.  
 Mechanistic Details of the Oscillatory Belousov-Zhabotinskii Reaction  
*J.Phys.Chem.* - 1990. - V.94. - №.18. - P. 7162-7170.

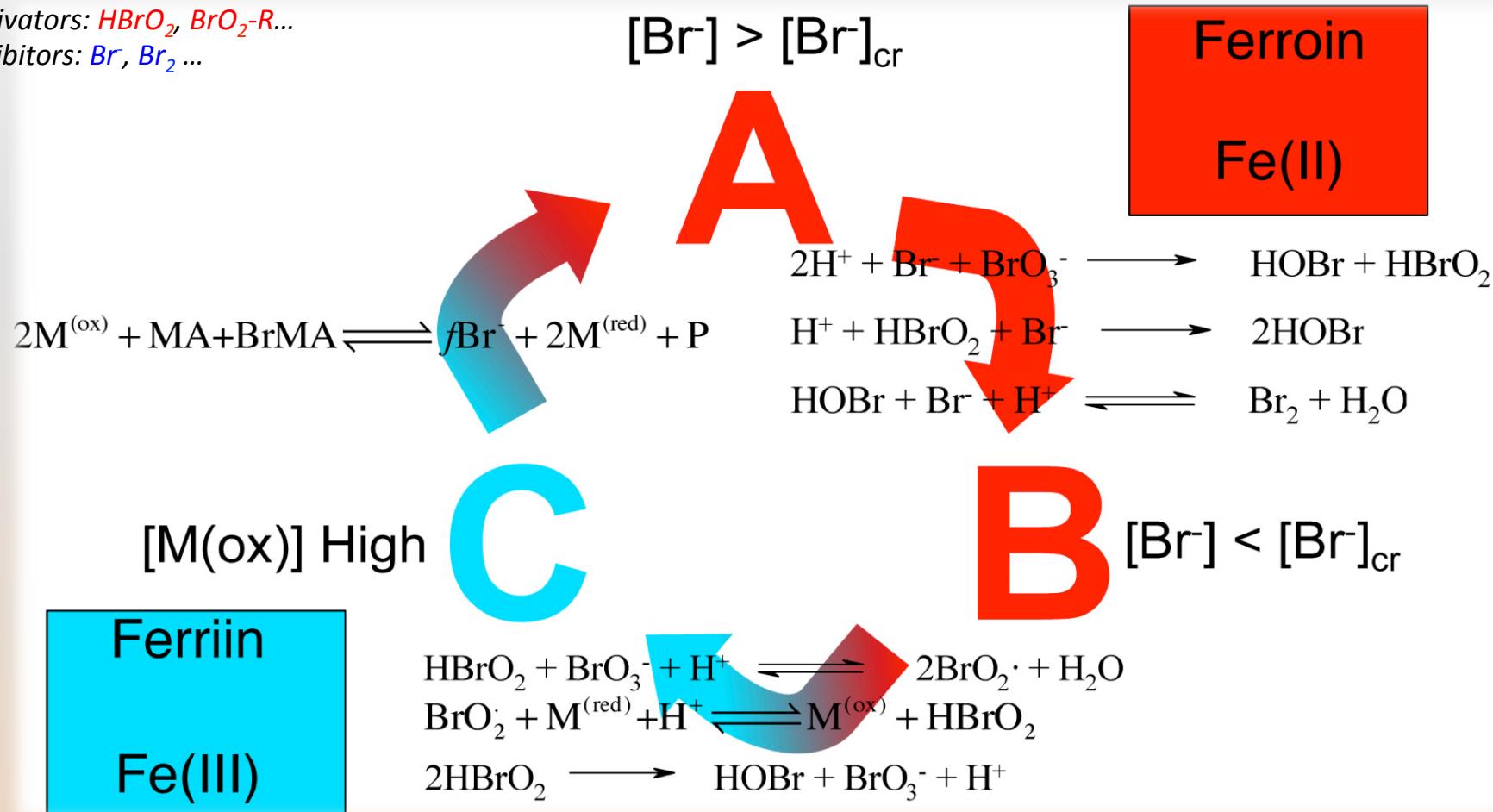
# The Belousov Zhabotinsky Reaction

## Oscillation Mechanism

oscillations originate from the alternation of inhibitory and excitatory steps.

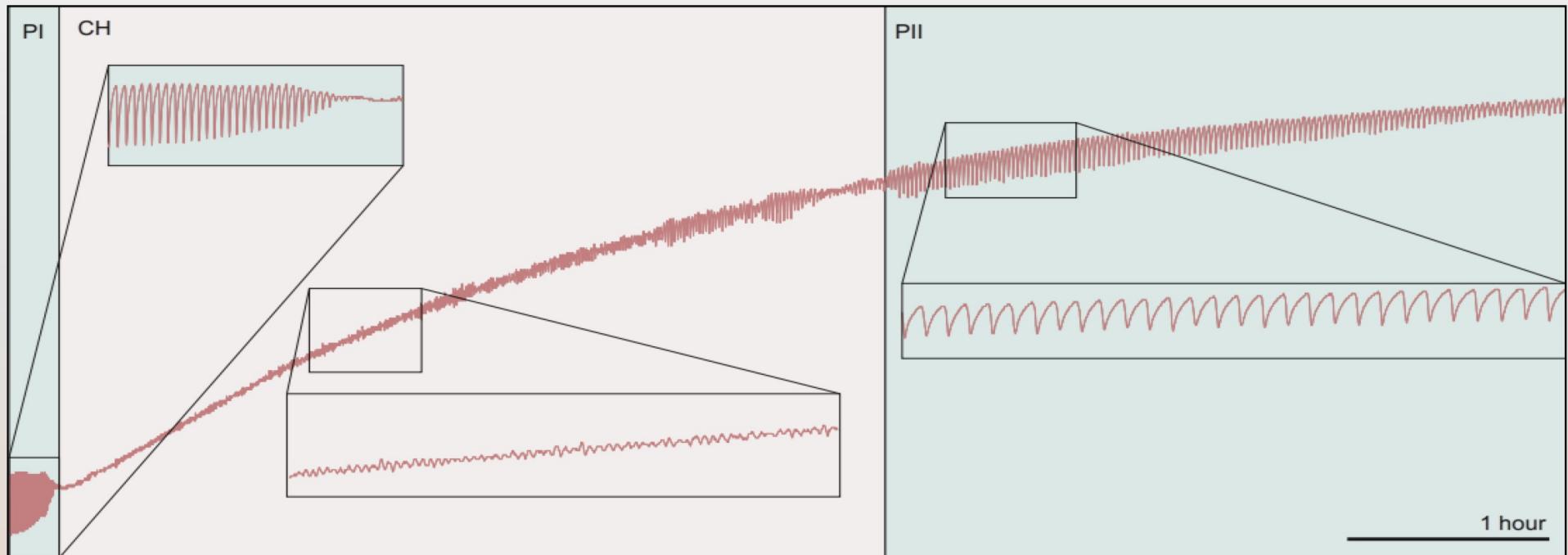
Activators:  $HBrO_2$ ,  $BrO_2\text{-}R\ldots$

Inhibitors:  $Br$ ,  $Br_2 \ldots$



# The Belousov Zhabotinsky Reaction

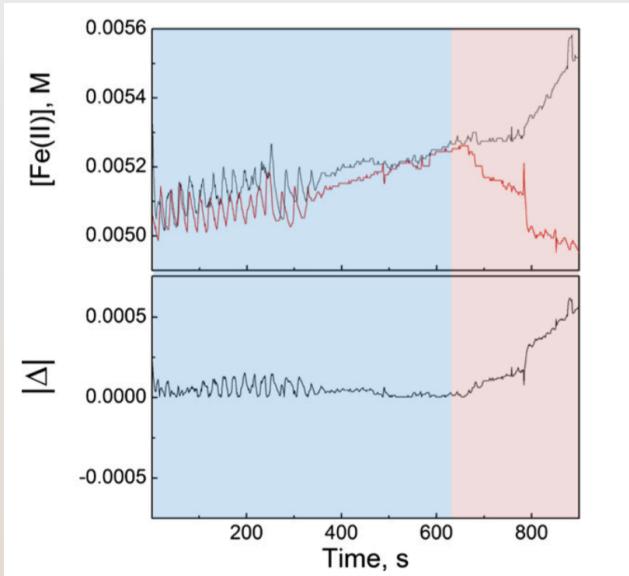
## *Long Time Behavior under non-stirring Conditions*



Photospectrometric recording of the transmittance at  $\lambda = 600 \text{ nm}$

# The Belousov Zhabotinsky Reaction

## *Deterministic Chaos in a Chemical System*



IOP Publishing

Eur. J. Phys. 35 (2014) 045005 (12pp)

European Journal of Physics

doi:10.1088/0143-0807/35/4/045005

### Butterfly effect in a chemical oscillator

M A Budroni<sup>1</sup>, F Wodlei<sup>1</sup> and M Rustici

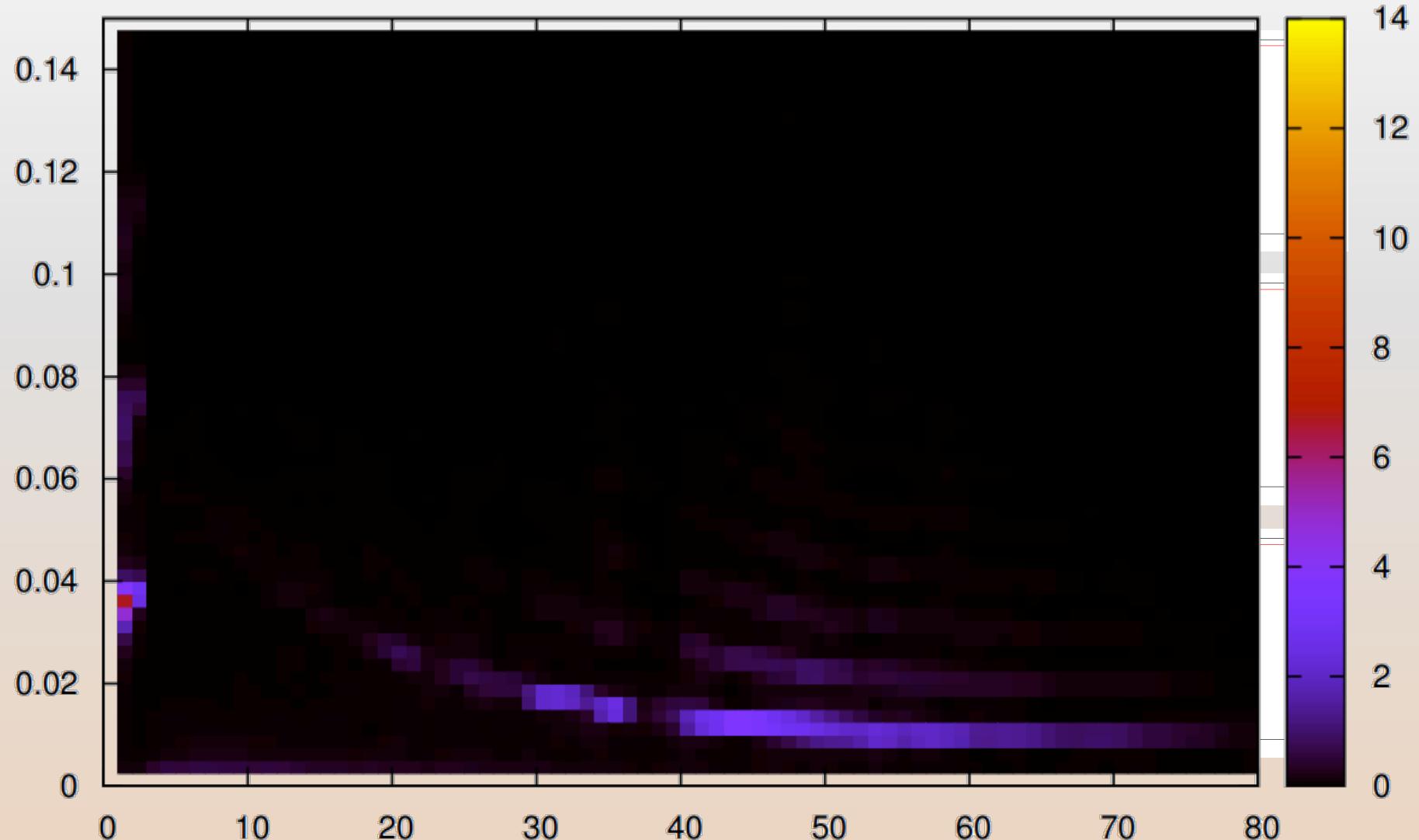
Dipartimento di Chimica e Farmacia, Università degli Studi di Sassari, Via Vienna 2,  
Sassari I-07100, Italy

*The Deviation of the transmittance signals of two “identic” solutions*

# The Belousov Zhabotinsky Reaction

## *Charactarisation of the Transient*

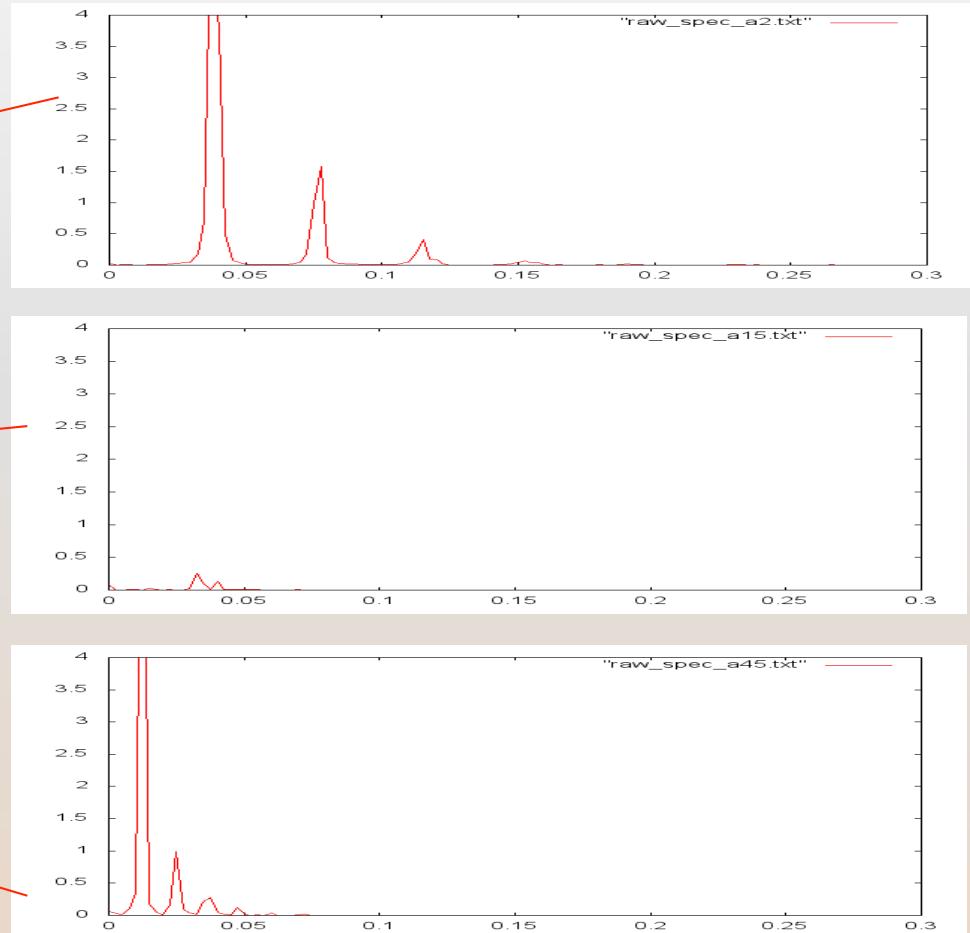
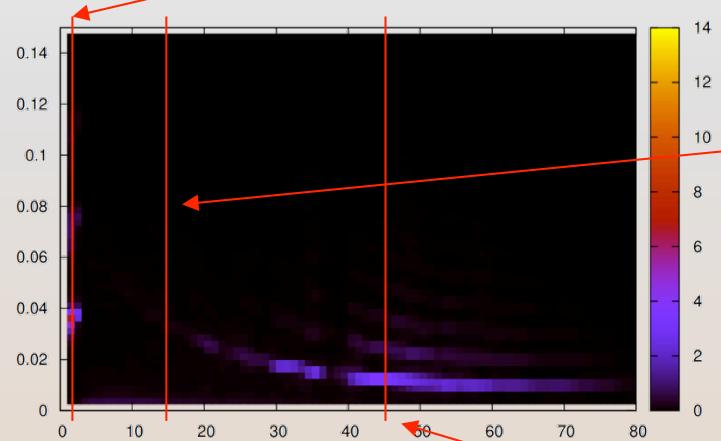
*“Dynamic” Fourier Spectra*



# The Belousov Zhabotinsky Reaction

## *Charactarisation of the Transient*

### ***Dynamic Fourier Spectra***



# The Belousov Zhabotinsky Reaction

## *The Chaotic Transient*

ELSEVIER

Chemical Physics Letters 263 (1996) 429–434

Evidence of a chaotic transient in a closed unstirred cerium catalyzed Belousov–Zhabotinsky system

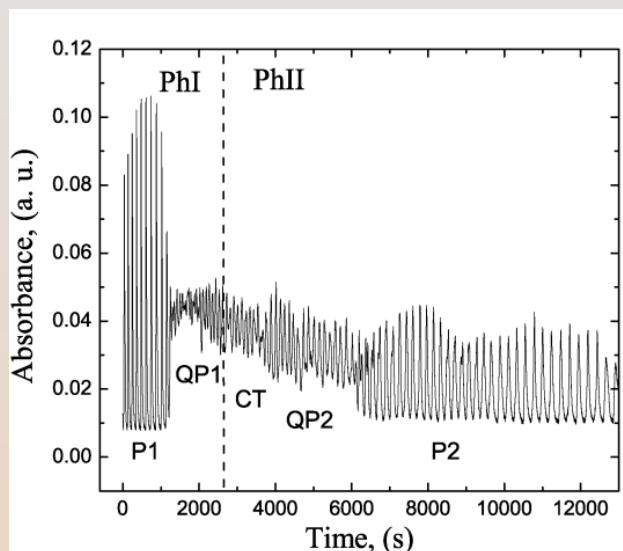
Mauro Rustici <sup>a,\*</sup>, Mario Branca <sup>a</sup>, Carlo Caravati <sup>a</sup>, Nadia Marchettini <sup>b</sup>

<sup>a</sup> Dipartimento di Chimica, Università di Sassari, Via Vienna 2, 07100 Sassari, Italy

<sup>b</sup> Dipartimento di Chimica, Università di Siena, Pian dei Mantellini 44, 53100 Siena, Italy

Received 1 August 1996; in final form 13 September 1996

period-1 (P1) -> quasiperiodicity (QP1) -> chaos (QT) = Ruelle-Takens-Newhouse scenario  
chaos (QT) -> quasiperiodicity (QP2) -> period-1 (P2) = *inverse* Ruelle-Takens-Newhouse scen.



Spectrophotometric time series at  $\lambda = 320$  nm of a cerium

ELSEVIER

Chemical Physics Letters 293 (1998) 145–151

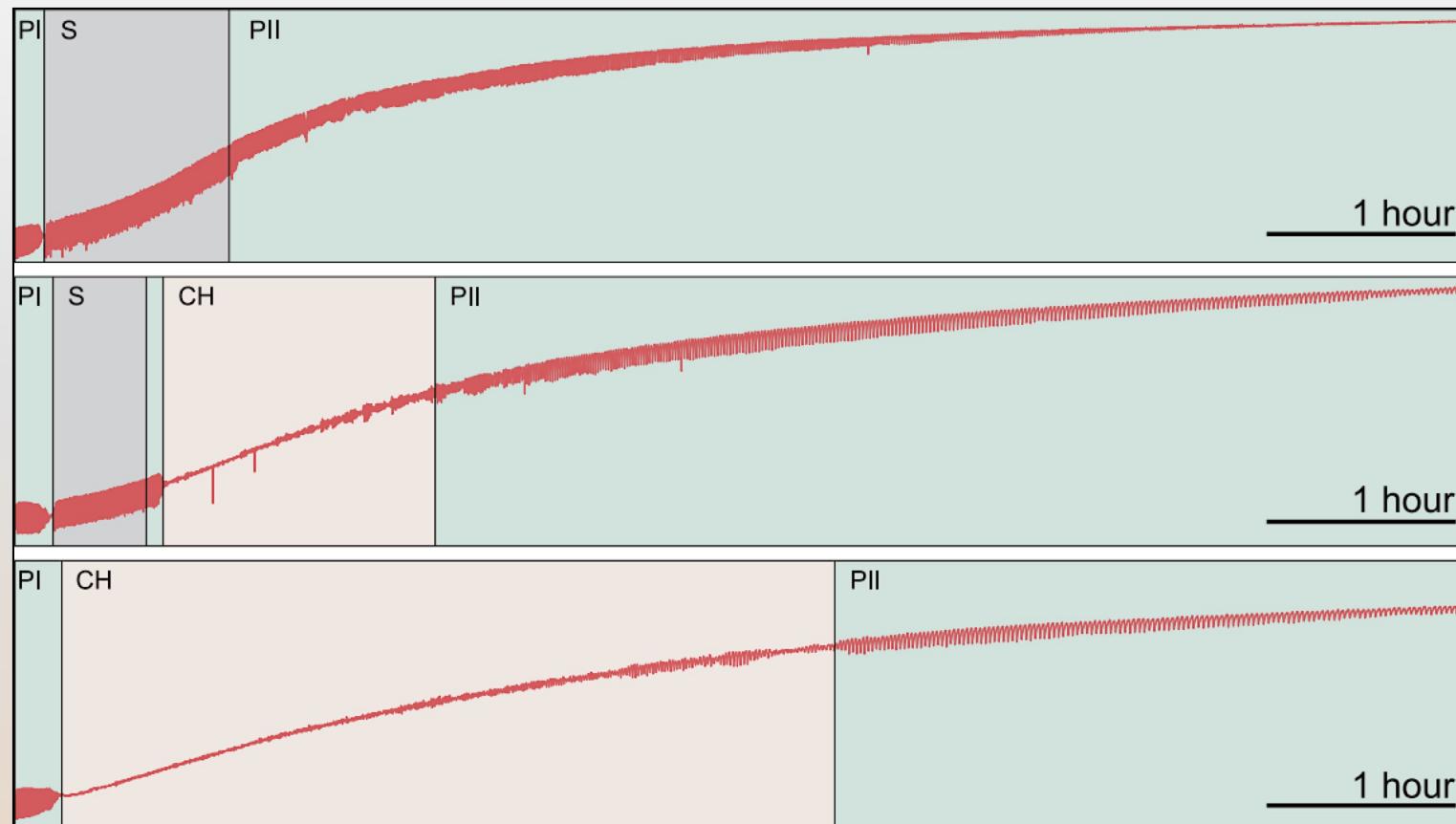
Inverse Ruelle–Takens–Newhouse scenario in a closed unstirred cerium-catalysed Belousov–Zhabotinsky system

Mauro Rustici <sup>a,\*</sup>, Mario Branca <sup>a</sup>, Antonio Brunetti <sup>b</sup>, Carlo Caravati <sup>a</sup>,  
Nadia Marchettini <sup>c</sup>

***Inverse Ruelle-Takens-Newhouse Scenario***

# The Belousov Zhabotinsky Reaction

## *Hydrodynamic Control of the Chaotic Transient*



Florian Wodlei, Mihnea R. Hristea

*Effect of Limited Stirring on the Belousov Zhabotinsky Reaction*

Proceedings of the European Conference on Complex Systems 2012 (September 2013) | ISBN: 978-3-319-00394-8

# The Belousov Zhabotinsky Reaction

## *Reaction Consumption and the Chaotic Transient*

The kinetic functions  $k_j(c_j, \bar{\lambda})$  are derived from the modified Oregonator model (see Appendix) and have the form

$$k_1(c_j, \bar{\lambda}) = \frac{dc_1}{d\tau} = \frac{1}{\varepsilon_1} \left( \frac{(qc_3 - c_1)}{(qc_3 + c_1)} fbc_2 + c_1(c_3 - c_1) \right) \quad (6)$$

$$k_2(c_j, \bar{\lambda}) = \frac{dc_2}{d\tau} = c_3c_1 - bc_2 \quad (7)$$

$$k_3(c_j, \bar{\lambda}) = \frac{dc_3}{d\tau} = -\frac{qc_3}{qc_3 + c_1} fbc_2 - c_1(c_1 - c_3) \quad (8)$$

where  $j = 1, 2, 3$ ,  $c_1$  is the concentration of bromous acid,  $c_2$  is the concentration of the oxidized form of the catalyst,  $c_3$  is the concentration of bromate and  $\bar{\lambda} = \varepsilon_1, q, f, b$  is the set of kinetic parameters.

PAPER

[www.rsc.org/pccp](http://www.rsc.org/pccp) | Physical Chemistry Chemical Physics

### Role of the reagents consumption in the chaotic dynamics of the Belousov–Zhabotinsky oscillator in closed unstirred reactors

Nadia Marchettini,<sup>a</sup> Marcello Antonio Budroni,<sup>\*a</sup> Federico Rossi,<sup>b</sup> Marco Masia,<sup>c</sup> Maria Liria Turco Liveri<sup>d</sup> and Mauro Rustici<sup>e</sup>

Received 1st April 2010, Accepted 8th June 2010

DOI: 10.1039/c0cp00109k

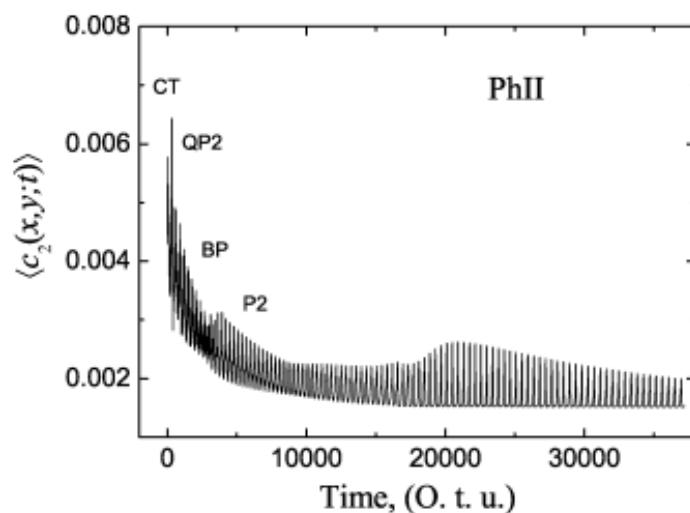


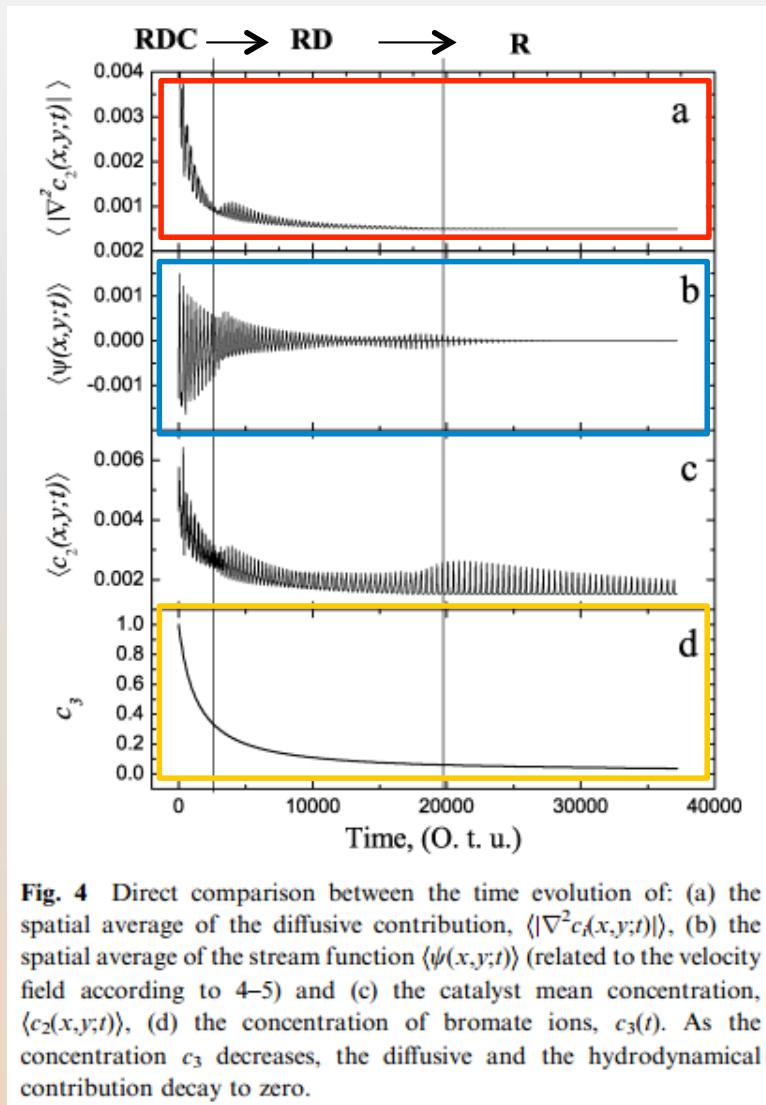
Fig. 2 Evolution of the catalyst ( $\langle c_2(x,y,t) \rangle$ ) in the region PhII.

### Chemical Kinetics vs. Diffusion vs. Convection

$$\frac{\partial c_i(x, y; t)}{\partial t} + D_\nu \left( u \frac{\partial c_i(x, y; t)}{\partial x} + v \frac{\partial c_i(x, y; t)}{\partial y} \right) - D_i \nabla^2 c_i(x, y; t) = k_i(c_j, \bar{\lambda}) \quad i, j = 1, 2, 3$$

# The Belousov Zhabotinsky Reaction

## *Reaction Consumption and the Chaotic Transient*



Chemical Kinetics vs. Diffusion vs. Convection

$c_2$  is the concentration of the oxidized form of the catalyst (e.g. Ferroin)

$c_3$  is the concentration of bromate ( $\text{BrO}_3^-$ )

**Fig. 4** Direct comparison between the time evolution of: (a) the spatial average of the diffusive contribution,  $\langle |\nabla^2 c_i(x,y,t)| \rangle$ , (b) the spatial average of the stream function  $\langle \psi(x,y,t) \rangle$  (related to the velocity field according to 4–5) and (c) the catalyst mean concentration,  $\langle c_2(x,y,t) \rangle$ , (d) the concentration of bromate ions,  $c_3(t)$ . As the concentration  $c_3$  decreases, the diffusive and the hydrodynamical contribution decay to zero.

# Our Group

## Living Systems Research

LIVING SYSTEMS  
RESEARCH

### Florian Wodlei

Theory and Experiment (Chemical Complexity and Self-Organization, Social Insects)

### Mihnea R. Hristea

Theory (Reaction-Diffusion-Convection, Graph Theory, Mathematical Statistics)

### Norbert Reichmann

Social Insects

### Jakob Klien

Student in Mathematics and Physics

### Christian Wodlei

Photography, Graphic-Design, Illustrations

10th Seminar (24/4/14) held at University Klagenfurt mit Lauro Langosco, Günther Pilz, Wilfried Elmenreich, Istvan Fehervari, Norbert Reichmann, Christian Wodlei, Jakob Klien, Mihnea Hristea und Florian Wodlei (von links)



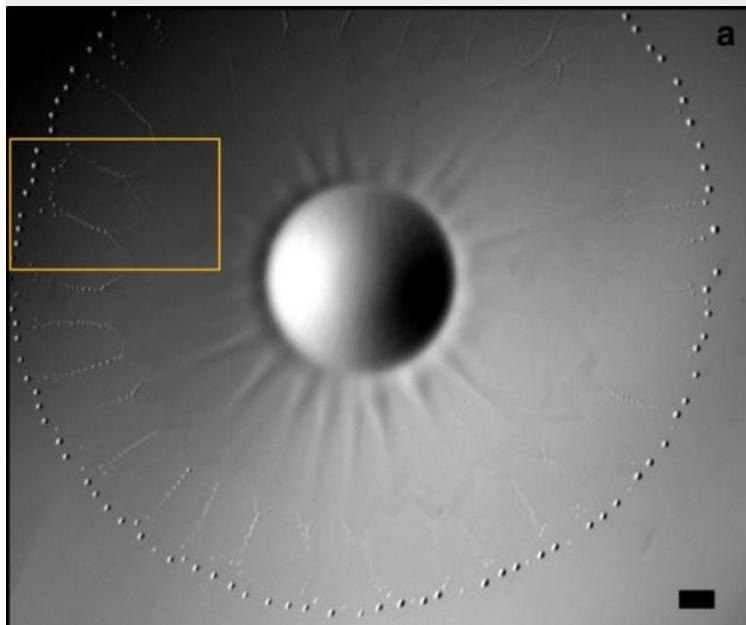
# Our Group

*Living Systems Research*

LIVING SYSTEMS

**RESEARCH**

*Chemical Self-Organization*



*Self-Organization of a Dichloromethane Droplet  
Belousov-Zhabotinsky Reaction*

F. Wodlei, J. Sebilleau, J. Magnaudet and V. Pimienta: *Marangoni-driven flower-like patterning of an evaporating drop spreading on a liquid substrate*. **Nature Communications** volume 9, 820 (2018)

*Social Insects*



*Carinthian Honey Bees  
Ants*

# Acknowledgements



**Marcello A. Budroni**

**Mauro Rustici**

Complexity and NonLinear Dynamics Group  
Department of Chemistry and Pharmacy.  
University of Sassari

**Federico Rossi**

Department of Chemistry and Biology  
University of Salerno



**Christian Bettstetter**

**Networked and Embedded Systems, Klagenfurt**

***Research coordinator Self-organizing Systems***

University of Klagenfurt

